mechanical properties of a solid propellant to a high degree can be readily removed on exposure to vacuum, and can also be reabsorbed upon re-exposure to atmosphere. I suggest that in addition to pre-exposure properties, the weight loss noted by the authors was substantially recovered upon re-exposure to the atmosphere. I also suggest that similar results could have been obtained had the specimens been tested in a desiccated environment.

### References

<sup>1</sup> Mugler, J. P., Jr. et al., "In Situ Vacuum Testing—A Must for Certain Elastomeric Materials," *Journal of Spacecraft and Rockets*, Vol. 6, No. 2, Feb. 1969, pp. 219–221.

<sup>2</sup> Muraca, R. F. and Whittick, J. S., "Polymers for Space-craft Applications," Project ASD-5046, JPL Contract 950745,

Sept. 15, 1967, Stanford Research Institute.

<sup>3</sup> Fishman, N., in "Space Environment Effects on Polymer Materials," Project ASD-4257, JPL Contract 950324, May 1965, Stanford Research Institute, pp. 37–50.

<sup>4</sup> Fishman, N., in "Polymers for Spacecraft Hardware: Materials Characterization," Project ASD-5046, JPL Contract 950745, Dec. 1966, Stanford Research Institute, pp. 51–73.

## Reply by Authors to N. Fishman

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THE main purpose of our Note was to show that vacuum-induced changes in the engineering properties of certain elastomeric materials must be determined by measuring the properties in the vacuum environment (in situ). This approach is in contrast to two other approaches in common use:

1) measuring the engineering properties before and after exposure to the vacuum environment, and/or 2) measuring the vacuum weight loss and assuming that the magnitude of the weight loss is indicative of changes in engineering properties. Our results showed that neither of the latter two approaches is valid for the materials studied.

The complete results of our study on the composite solid propellant are presented in Ref. 1, which describes a phenomenological model for the behavior. The analysis¹ indicates that vacuum exposure removes interfacial moisture that results in the observed property changes. Thus, our results are in accord with Fishman's comments on moisture effects. Reference 1 also describes tests of samples in a desiccated environment (dry nitrogen) and the results show that, at a given storage time, the changes in mechanical properties for the samples stored in vacuum were substantially greater than for the samples stored in a desiccated environment.

The authors recommend that in situ engineering properties measurements be used to evaluate spacecraft materials rather than weight loss or other peripheral measurements.

## Reference

<sup>1</sup> Greenwood, L. R., "The Effect of Vacuum on the Mechanical Properties of a Solid Rocket Propellant During Space Storage," thesis, May 1967, Virginia Polytechnic Institute, Blacksburg, Va.; University Microfilms, Order 68-2047.

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# Comments on "Wobble-Spin Technique for Spacecraft Inversion and Earth Photography"

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ALTHOUGH the results of a recent paper¹ by Beachley and Uicker are correct for the assumptions made, it should be emphasized that the concept cannot be reasonably implemented for the stated application. The paper clearly states that the wobble-spin technique is a workable concept "if certain prescribed spacecraft moment-of-inertia relationships are maintained" but then fails to consider the devastating effect of a small deviation from those mass property constraints. It is instructive, in terms of feasibility, to consider the performance of the proposed system in the presence of a small deviation from the idealized mass properties. Their Eqs. (A11–A13) are used as published;

$$I_{11}\dot{\omega}_1 = -J_0\dot{\Omega} + (I_{22} - I_{33})\omega_2\omega_3 \tag{A11}$$

$$I_{22}\dot{\omega}_2 = -J_0\Omega\omega_3 - (I_{11} - I_{33})\omega_1\omega_3$$
 (A12)

$$I_{33}\dot{\omega}_3 = J_0\Omega\omega_2 + (I_{11} - I_{22})\omega_1\omega_2 \tag{A13}$$

Instead of rewriting the equations at once with the oversimplifying constraint  $I_{22} = I_{33}$  (as in Ref. 1), it is better to solve the general equations for small deviations from the initial conditions. Consider the wheel accelerating period to be very short, after which the wheel runs at constant speed  $\Omega$ specified by their Eq. (A23). The unsymmetrical spinning spacecraft may be conveniently analyzed by introducing a factor k as in Ref. 2, where

$$k^2 = I_{22}(I_{22} - I_{33})/I_{11}(I_{11} - I_{33})$$

and

$$\omega = \omega_1 + ik\omega_2$$

Then, considering  $|\omega| \ll 1$  and  $\omega_1 t \ll 1$ , the  $e_3$  axis remains near the angular momentum vector and Eqs. (A11-A13) reduce to

$$\dot{\omega}_1 - \Omega_1 \omega_2 = -J_0 \dot{\Omega} / I_{11}$$

$$\dot{\omega}_2 + \Omega_2 \omega_1 = -J_0 \Omega p / I_{22}$$
(1)

where

$$\Omega_1 = p \left( \frac{I_{22} - I_{33}}{I_{11}} \right) \quad \Omega_2 = p \left( \frac{I_{11} - I_{33}}{I_{22}} \right)$$

$$p = \omega_3 \approx \text{const}$$

Equations (1) become

$$\dot{\omega} + i\Omega_n \omega = -\dot{h}/I_{11} - ikhp/I_{22} \tag{2}$$

where  $\Omega_n = (\Omega_1 \Omega_2)^{1/2}$ , and  $h = J_0 \Omega =$  angular momentum of reaction wheel. The solution of Eq. (2) for  $t < t_1$  (where  $t_1 =$  wheel acceleration period) is

$$\omega = \omega(0)e^{-i\Omega_n t} + i(1 - e^{-i\Omega_n t})(1/I_{11}\Omega_n - kp/I_{22}\Omega_n^2)\dot{h} - (kp\dot{h}/I_{22}\Omega_n)t$$
(3)

The nutation frequency  $\Omega_n$  is near zero, so that  $\Omega_n t_1 \ll 1$  (where  $t_1$  is the wheel accelerating period;  $t_1 \ll 1$ ), and with  $\omega(0) = 0$  the spacecraft motion at the end of the wheel accelerating period is

$$\omega(t_1) \approx -\dot{h}t_1/I_{11} = -J_0\Omega/I_{11}$$
 (4)

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\* Manager, Guidance and Dynamics Department, Space Systems Division. Member AIAA. which, of course, is the authors' result [Eq. (A15)], because  $\Omega_n \approx 0$  is equivalent to  $I_{22} \approx I_{33}$ . However, returning to Eq. (2), with h = 0 and using (4) as the initial conditions, we have

$$\omega(t) = (-h/I_{11} + khp/I_{22}\Omega_n)e^{-i\Omega_n t} - khp/I_{22}\Omega_n$$
 (5)

which expands to

$$\omega_1(t) = h \left( \frac{kp}{I_{22}\Omega_n} - \frac{1}{I_{11}} \right) \cos\Omega_n t - \frac{khp}{I_{22}\Omega_n}$$
 (6)

$$\omega_2(t) = h(1/kI_{11} - p/I_{22}\Omega_n) \sin\Omega_n t \tag{7}$$

The system performance should now be evaluated on the basis of Eqs. (6) and (7) with  $k \to 0$ ,  $\Omega_n \to 0$ , rather than the authors' Eqs. (A18) and (A21). Using the Euler angles of Fig. 1 to define the inertial motion of the body axes, the angles of interest (for small angles) are given by

$$\dot{\alpha} \approx \omega_1 + \beta p \qquad \dot{\beta} \approx \omega_2 - \alpha p \tag{8}$$

Substituting (6) and (7) in Eqs. (8) with  $\alpha(0) = \beta(0) = 0$  and  $\dot{\alpha}(0) = -h/I_{11}$ , the solution for the tilt angle  $\alpha$  and the cant angle  $\beta$  is

$$\alpha = -h \left( \sin \Omega_n t \right) / I_{11} \Omega_n \tag{9}$$

$$\beta = h (1 - \cos\Omega_n t) / p(I_{11} - I_{33}) \tag{10}$$

Equations (9) and (10) define the deviation of the  $e_3$  axis from the angular momentum vector. In the limit as  $\Omega_n \to 0$  (i.e.,  $I_{22} \to I_{33}$ ), Eq. (9) reduces to the authors' result  $\alpha = -ht/I_{11}$  as given by Eq. (A23). But Table 1 clearly shows that the concept cannot be reduced to practice. The authors' values are used, with  $\omega_0 = p = 10.46$  rad/sec,  $J_0\Omega = h = 0.0225$  ft-lb-sec, and  $I_{11} = 124$  slug-ft².  $I_{33}/I_{11}$  is taken to be 1.4.

It is clear that the  $e_3$  axis never departs very far from the angular momentum vector even when  $I_{22}$  and  $I_{33}$  differ by less than one part in a million. The requirements on  $I_{22}$  and  $I_{33}$  for  $\alpha$  to reach the desired 15° are

$$1 \ge I_{22}/I_{33} > 0.999999999$$
 (approximately) (11)

Also, the cant angle  $\beta$  is not zero and must be accounted for. For example,  $\beta_{\rm max}=2h/[p(I_{33}-I_{11})]$ , which for the preceding example is 0.005°, a value probably unacceptable for the 2400-line, 15° picture.

There are some other areas of difficulty in the proposed technique (i.e., alignment, thermal distortion, and residual excess energy). Since there is no preferred spin axis, any finite amount of excess energy in the system will result in an unwanted drift about the  $e_1$  axis. The difficulty inherent in stabilizing a spinning body having two nearly equal moments of inertia is analyzed on an energy dissipation basis in Ref. 3.

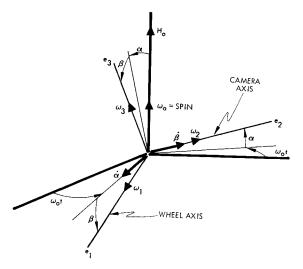


Fig. 1 Euler angles defining spacecraft motion.

Table 1 Maximum available tilt angle,  $\alpha_{max}$ 

$I_{22}/I_{33}$	$\Omega_n$ , rad/sec	$\alpha_{ m max}$ , deg
0.99	0.66	0.0158
0.999	0.209	0.0498
0.9999	0.066	0.158
0.99999	0.0209	0.498
0.999999	0.0066	1.58

In summary, it may be said that the "practical method of control" referred to in the authors' footnote<sup>1</sup> must be directed toward the inequality (11) which leaves little for the engineer to work with when considering the proposed technique.

#### References

<sup>1</sup> Beachley, N. H. and Uicker, J. J., Jr., "Wobble-Spin Technique for Spacecraft Inversion and Earth Photography," *Journal of Spacecraft and Rockets*, Vol. 6, No. 3, March 1969, pp. 245–248

<sup>2</sup> Grasshoff, L. H., "An Onboard, Closed-Loop Nutation Control System for a Spin-Stabilized Spacecraft," *Journal of Spacecraft and Rockets*, Vol. 5, No. 5, May 1968, pp. 530–535.

<sup>3</sup> Kuebler, M. E., "Gyroscopic Motion of an Unsymmetrical Satellite Under No External Forces," TN D-596, Dec. 1960, NASA.

# Reply by Authors to L. H. Grasshoff

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THE point raised by L. H. Grasshoff is very significant. His analysis shows the extreme mechanical accuracy necessary for the example of Ref. 1 to work in the manner indicated, and should contribute to a better understanding of systems of this type. Our computer simulation results, from continuing development of this concept, agree with the values presented by Grasshoff. We must agree, therefore, that the concept in its pure form is indeed very sensitive to small errors in the  $I_{22}/I_{33}$  ratio (nominally unity), much more so than one would like.

The numbers presented by Grasshoff in his Table 1 all apply to the "stable" case, with  $I_{33} > I_{22}$ . It is equally important, in working out the details of a practical system, to consider the "unstable" case with  $I_{33} < I_{22}$ . (Since we also have  $I_{33} > I_{11}$ , this corresponds to the condition of "unstable equilibrium" of a rigid body rotating about its principal axis of intermediate moment of inertia.) With this unstable condition, the tilt rate  $\dot{\alpha}$  will increase as the tilt angle  $\alpha$  increases (up to 90°), the amount of this increase depending upon degree of instability. We will discuss individually the two applications: a) inversion of a spin-stabilized satellite, and b) spin-scan earth photography with the camera rigidly attached to the basic satellite structure.

## **Spacecraft Inversion**

For inversion of a spin-stabilized satellite, one probably is not concerned with the exact tilt rate or the degree of nutation during the maneuver itself, but merely wants a quick, reliable, low-energy-consumption technique. A larger flywheel could overcome a practical error tolerance in the  $I_{22}/I_{33}$  ratio and reduce inversion time, but the flywheel's size and power consumption would no longer be insignificant. Probably a better approach is to have the spacecraft slightly stable during normal operation, but changed (as by a small translating

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